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Option pricing method based on partial differential brownian berg solution model

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Abstract. In order to improve the accuracy of the pricing problem about Down-and-Out discrete barrier option and reduce the computational complexity, a kind of Romberg solution with barrier option partial differential Brownian mode of discrete time parameter is proposed. First of all, the Down-and-Out discrete barrier option problem is modeled as a geometric Brownian motion model with the parameters changing over time, and the option pricing of partial differential equation (PDE) is performed by using corresponding time change not related to the time. Then the time-independent PDE obtained is transformed into a simple heat conduction equation form to realize the model simplification and give the theorem of discrete barrier option pricing. Finally, an accurate solving of discrete barrier option Brownian model is performed by using Romberg solving process. The effectiveness of proposed method is verified by the numerical test result.

Key words. Time parameter, Option pricing, Discrete barrier option, Partial differential equation (PDE), Brownian model.

1. Introduction

The international and domestic financial markets are changing with each passing day in recent years. For different market demands, financial institutions have put forward different new options. Barrier option is one of them, also called threshold option [1, 2]. The option income depends on the level at which the underlying asset price has reached in a particular period. Most types of barrier options are traded outside the market, which is generally lower than the price of the conventional option, but it can reduce the risk in financial investments. At the same time, the barrier option in investment project strategy can effectively meet the specific demands of investors, allow investors to wait or give up the investment projects according to their habits and carry out other investment projects timely, so it is concerned by many market participants. Barrier options are generally divided knock-in and knock-out

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options [3, 4]. The former is the underlying asset price reaching a certain barrier, then options exist, and the latter is just the opposite. As to American barrier option, it is difficult to obtain the closed loop solving due to its greater pricing difficulty caused by the indeterminacy of share price and time routing, and it is generally solved by approximate pricing with numerical method.

Classic numerical methods for option pricing include Monte Carlo and tree graph methods [5, 6], and the later contains binary tree, ternary tree and so on. At present, the main barrier option pricing method is the calculation method based on the tree graph, but such method exists many problems, which are mainly concentrated in convergence. So there is a very large calculation error [7]. Such error is caused by the differences between real barrier and tree graph barrier. The financial practice shows that the underlying asset pricing will show occasional "jump" phenomenon, which is mainly caused by unexpected events, such as human speculation, policy adjustment, bankruptcy, etc., which can lead to a sudden drop in asset pricing. In order to overcome the above problems, a limiting method for option pricing of binary tree is designed in Literature [8], although this method realizes the approximate convergence of the underlying asset and option barriers, the closer the level between price and barrier is, the longer the convergence time of computation is. The decomposed barrier option pricing technique can solve the shortcomings of the pricing of tree graphs. Such method is a fast computational method but has many constraints, which is not convenient for practical application. The Monte Carlo option pricing method based on least square algorithm is designed in Literature [9]. It is proved that such method can reduce the error by using sequence certainty to replace pseudo random sequence in low dimension. Although it can solve the problem of dependent option pricing and overcome the barrier problem by using interpolation method, some problems also exist in the convergence of such method.

2. Description of discrete barrier option model

The foreign call option pricing is equal to the simple European option pricing, so only one of them needs to be studied , and other kinds of barrier options, such as put options, can be priced with buying call options. Assumed that the price of underlying stocks can be expressed by a random process X_t. Follow the process of Geometric Brownian Motion (GBM) [10, 11]:

$$dX_t = (\rho(t) - D(t))X_t dt + \sigma(t)X_t dW_t^Q \,, \tag{1}$$

in which $X(0) = x_0$; the random process W_t^Q is the standard Brownian Motion in the case of risk-neutral measure Q and deterministic function $\rho(t)$, which is the risk-free interest rate changing over time; $D(t)$ is the unit dividend in a certain time; $\sigma(t)$ is time-dependent momentary fluctuation. Considering the monotonically increased monitoring date, the form is:

$$0 = t_0 < t_1 < \cdots < t_n < \cdots < t_N = T \,. \tag{2}$$

Assumed that L and E respectively are the lower barrier price and executive price. Meanwhile $P_b = P_b\,(x, t, n)$ represents call option price; variable x represents the share price in the time period $t \in [t_n, t_{n+1}]$. According to the famous *Black–Scholes* PDE [12, 13], the option price meets following relational expression:

$$-\frac{\partial P_b}{\partial t} + (\rho(t) - D(t))x\frac{\partial P_b}{\partial x} + \frac{1}{2}\sigma^2(t)x^2\frac{\partial^2 P_b}{\partial^2 x} - \rho(t)P_b = 0 \tag{3}$$

The conditions are as follows:

$$\begin{cases} P_b(x, T, 0) = (x - E)1_{(x \geq \max(E,L))}, n = N \\ P_b(x, t_{N-n}, N - n - 1) = P_b(x, t_{N-n}, N - n)1_{(x \geq L)}, n = 1, 2, \cdots, N - 1 \end{cases}$$

The paper mainly studies the unknown option price in the case of initial time $t = 0$. The option price of zero time must be obtained by solving reversed PDE from the perspective of the option payment condition of final due day T. In order to find discrete barrier option price (basing on the definitions of up and down barrier options), following initial condition must be provided in every monitoring data:

$$\begin{cases} P_b(x, t_0, 0) = (x - E)1_{(x \geq \max(E,L))}, n = 0 \\ P_b(x, t_n, n) = P_b(x, t_n, n - 1)1_{(x \geq L)}, n = 1, 2, \cdots, N - 1 \end{cases} \tag{4}$$

in which $\mathbf{1}_{(x \geq L)}$ is characteristic function; when $x \geq L$, $\mathbf{1}_{(x \geq L)} = 1$; when $x < L$, $\mathbf{1}_{(x \geq L)} = 0$. $P_b(x, t_n, n)$ can be defined as:

$$P_b(x, t_n, n - 1) := \lim_{t \to t_n} -P_b(x, t, n - 1)\,. \tag{5}$$

Convert the time-varying parameter in Formula (3) to a fixed parameter. On such basis, respectively give the variations of option price, share price and time as follows:

$$\begin{cases} P_b(x, t, n) = h_n(t)\bar{P}_b(\bar{x}, \bar{t}, n)\,, \\ \bar{x} = \phi_n(t)x, \bar{t} = \psi_n(t)\,, \end{cases} \tag{6}$$

in which function h_n, ϕ_n and ψ_n are unknown. Basing on these functions, PDE (3) can be transformed into constant coefficient PDE in every monitoring interval. The forms of $\frac{\partial P_b}{\partial t}$, $\frac{\partial P_b}{\partial x}$ and $\frac{\partial^2 P_b}{\partial x^2}$ is given as follows by using chain rules:

$$\frac{\partial P_b}{\partial t} = \frac{\partial h_n(t)\bar{P}_b}{\partial t} = h_n^r(t)\bar{P}_b + h_n(t)(\frac{\partial \bar{P}_b}{\partial \bar{t}}\psi_n^r(t) + \frac{\partial \bar{P}_b}{\partial \bar{x}}\phi_n^r(t)x)\,.$$

$$\frac{\partial P_b}{\partial x} = \frac{\partial (h_n(t)\bar{P}_b)}{\partial x} = h_n(t)\phi_n(t)\frac{\partial \bar{P}_b}{\partial \bar{x}}\,.$$

$$\frac{\partial^2 P_b}{\partial x^2} = \frac{\partial^2 (h_n(t)\bar{P}_b)}{\partial x^2} = h_n(t)\phi_n(t)^2\frac{\partial^2 \bar{P}_b}{\partial \bar{x}^2}\,.$$

PDE (3) is rewritten as follows by using above variables:

$$\begin{aligned} &-\frac{\partial \bar{P}_b}{\partial \bar{t}}+(\frac{-\phi_n^r(t)+(p(t)-D(t))\phi_n(t)}{\psi_n(t)}x)\frac{\partial \bar{P}_b}{\partial \bar{x}} \\ &+\frac{\sigma^2(t)\phi_n(t)^2}{2\psi_n^r(t)}x^2\frac{\partial^2 \bar{P}_b}{\partial \bar{x}^2}=\bar{P}_b(\frac{h_n^r(t)}{h_n(t)\psi_n^r(t)}+\frac{P(t)}{\psi_n(t)})\,, \end{aligned} \tag{7}$$

If the coefficient of above constant equation needs to be obtained, ρ_n and σ_n meets following equations:

$$\begin{aligned} &P_n\bar{x}=\frac{-\phi_n'(t)+(\rho(t)-D(t))\phi_n(t)}{\psi_n'(t)}x \\ &\frac{1}{2}\sigma_n^2\bar{x}^2=\frac{1}{2}\frac{\sigma^2(t)\phi_n(t)^2}{\psi_n'(t)}x^2, \rho_n=\frac{h_n'(t)}{h_n(t)\psi_n'(t)}+\frac{\rho(t)}{\psi_n'(t)} \end{aligned} \tag{8}$$

As to above equation solving, it can be obtained as follows:

$$\begin{cases} \phi_n(t)=B_n\exp\left(\displaystyle\int_{t_n}^{t}((\rho(u)-D(u))-\rho_n\psi_n'(u))\,du\right), \\ \psi_n(t)=\dfrac{1}{\sigma_n^2}\displaystyle\int_{t_n}^{t}\sigma^2(u)\,du+A_n\,, \\ h_n(t)=C_n\exp\displaystyle\int_{t_n}^{t}(\rho_n\psi_n'(u)-\rho(u))\,du\,. \end{cases} \tag{9}$$

Therefore, the general form of function $h_n(t)$, $\psi_n(t)$ and $\phi_n(t)$ can be obtained in monitoring interval $[t_n, t_{n+1}]$. PDE (3) can be transformed into following PDE through such transformation:

$$-\frac{\partial \bar{P}_b}{\partial \bar{t}}+\rho_n\bar{x}\frac{\partial \bar{P}_b}{\partial \bar{x}}+\frac{1}{2}\sigma_n^2\bar{x}^2\frac{\partial^2 \bar{P}_b}{\partial \bar{x}^2}-\rho_n\bar{P}_b=0\,, \tag{10}$$

in which $n=0,1,\cdots,N-1$; select new constant A_n, B_n, C_n, ρ_n and σ_n to satisfy condition (4). Firstly try to determine the fixed value A_0, B_0 and C_0, then $\phi_0(t_0)=1$ can be obtained by considering $B_0=1$ according to Definition (9), thus:

$$\begin{aligned} &(\bar{x}-E)D1_{(x}\geq\max(E,L))= \\ &(\phi_n(t_0)x-E)1_{(\phi_n(t_0)x\geq\max(E,L))}=(x-E)1_{(x\geq\max(K,L))}\,. \end{aligned} \tag{11}$$

Set $C_0=1$ according to Formula (9), following equation can be obtained:

$$P_b(\bar{x},\bar{t}_0,0)=\frac{P_b(x,t_0,0)}{h_0(t_0)}=P_b(x,t_0,0)\,. \tag{12}$$

Considering $A_0=0$, $\bar{t}_0=\psi_0(t_0)=0$ can be obtained. The starting point of $\bar{t}$ is consistent with time variable t. Then determine constant A_n, B_n and C_n. Thus Formula (6) can be obtained according to the Formula (4), and follows can be obtained after modifying the left side of Formula (6):

$$P_b(\bar{x},\bar{t}_n,n)=\frac{P_b(x,t_n,n)}{h_n(t_n)},\bar{x}=\phi_n(t_n)x,\bar{t}_n=\psi_n(t_n)\,. \tag{13}$$

Follows can be obtained after modifying the right side of Formula (6):

$$\begin{cases} P_b(\bar{x},\bar{t}_n,n-1)1_{(x\geq\, L)}=\dfrac{P_b(x,t_n,n-1)}{h_{n-1}(t_n)}1_{(\phi_{n-1}(t_n)x\geq L)}\,, \\ \bar{x}=\phi_{n-1}(t_n)x,\bar{t}_n=\psi_{n-1}(t_n)\,. \end{cases} \tag{14}$$

Therefore, the required equivalency can be guaranteed if considering the below assumptions of $h_n(\cdot)$, $\phi_n(\cdot)$ and $\psi_n(\cdot)$.

$$\begin{cases} \phi_{n-1}(t_n)=\phi_n(t_n)=1\,, \\ \psi_{n-1}(t_n)=\psi_n(t_n)\,, \\ h_n(t_n)=h_{n-1}(t_n)\,. \end{cases} \tag{15}$$

3. discrete barrier option pricing

3.1. Main conclusion of study

Theorem 1: if considering follows in every monitoring interval:

$$\begin{cases} A_n=t_n; n=1,2,\cdots,N-1 \\ \sigma_n^2=\dfrac{1}{t_{n+1}-t_n}\displaystyle\int_{t_n}^{t_{n+1}}\sigma^2(u)du, n=0,1,\cdots,N-1\,. \end{cases} \tag{16}$$

Then $\psi_{n-1}(t_n)=\psi_n(t_n)$ is satisfied.
Prove: follows can be obtained by using the definition of $\psi_n(\cdot)$:

$$\begin{aligned} \psi_{n-1}(t_n)&=\frac{1}{\sigma_{n-1}^2}\int_{t_{n-1}}^{t_n}\sigma^2(u)du+A_{n-1}=\frac{\int_{t_{n-1}}^{t_n}\sigma^2(u)du}{\frac{1}{t_n-t_{n-1}}\int_{t_n-1}^{t_n}\sigma^2(u)du}+t_{n-1} \\ &=t_n=\frac{1}{\sigma_n^2}\int_{t_n}^{t_{n+1}}\sigma^2(u)du+A_n=\psi_n(t_n)\,. \end{aligned} \tag{17}$$

Completed!
Thus as to every monitoring date, value σ_n^2 can be considered as the average value of the fluctuation square of such interval.
Theorem 2: considering follows for every monitoring interval:

$$\begin{cases} B_n=1; n=1,2,\cdots,N-1 \\ \rho_n=\dfrac{1}{t_{n+1}-t_n}\displaystyle\int_{t_n}^{t_{n+1}}(\rho(u)-D(u))du, n=0,1,\cdots,N-1 \end{cases} \tag{18}$$

Then $\phi_{n-1}(t_n) = \phi_n(t_n) = 1$ is satisfied. Moreover, as to $h_n(t_n) = h_{n-1}(t_n)$, the form of constant C_n is:

$$C_n = C_{n-1}\exp(\int_{t_n}^{t_{n+1}} ((\rho(u) - D(u)) - \rho_n \psi'_n(u))du)\,. \tag{19}$$

In which $n = 1, 2, \cdots, N-1$.

Prove: basing on function $\psi_n(\cdot)$ and $\phi_n(\cdot)$, follows can be obtained:

$$\begin{aligned}\phi_{n-1}(t_n) &= B_{n-1}\exp(\int_{t_{n-1}}^{t_n} ((\rho(u) - D(u)) - \rho_{n-1}\psi'_{n-1}(u))du \\ &= \exp(\int_{t_{n-1}}^{t_n} (\rho(u) - D(u))du - \int_{t_{n-1}}^{t_n} (\rho(u) - D(u))du) = 1\,.\end{aligned} \tag{20}$$

Thus:

$$\phi_n(t_n) = B_n \exp((\int_{t_n}^{t_{n+1}} (\rho(u) - D(u)) - \rho_n\psi'_n(u))du)du) = 1\,. \tag{21}$$

Similarly the section part of Theorem 2 can be proved according to the definition of $h_n(t)$. Completed!

as previously mentioned, ρ_n is selected as the average value of $\rho(t) - D(t)$ in the nth monitoring interval $[t_n, t_{n+1}]$, PDE (3) and initial condition (4) can be transformed into PDE (10). Note that the monitoring date, share price and option price is unchanged in such transformation. In order to obtain $\bar{P}_b(\bar{x}, T, N-1)$, PDE (10) shall be solved. Thus following conventional transformation is performed in every monitoring interval.

$$\begin{cases} \bar{P}_b(\bar{x}, \bar{t}, n) = W(z, \bar{t}, n)\,, \\ z = \ln(\dfrac{\bar{x}}{L}), k = \ln(\dfrac{E}{L})\,. \end{cases} \tag{22}$$

Follows can be obt ained after rewiring PDE (10) by using $W(z, \bar{t}, n)$:

$$-\frac{\partial W}{\partial \bar{t}} + m_n \frac{\partial W}{\partial z} + \frac{\sigma_n^2}{2}\frac{\partial^2 W}{\partial z^2} - \rho_n W = 0\,. \tag{23}$$

then $m_n = \rho_n - \frac{\sigma_n^2}{2}$ is obtained, and following conditions can be known:

$$\begin{cases} W(z, \bar{t}_0, 0) = L(e^z - e^k)1_{(z\geq\delta)}\,, \\ W(z, \bar{t}_n, n) = W(z, \bar{t}_n, n-1)1_{(z\geq 0)}\,, \end{cases} \tag{24}$$

in which $\delta = \max\{k, 0\}$, $n = 1, 2, \cdots, N-1$. For every monitoring interval, the further transformation is as follows:

$$W(z, \bar{t}, n) = e^{\alpha_n z + \beta_n \bar{t}} g(z, \bar{t}, n), n = 0, 1, \cdots, N-1 \tag{25}$$

α_n and β_n can be defined as follows:

$$\alpha_n = -\frac{m_n}{\sigma_n^2}, \beta_n = \alpha_n m_n + \alpha_n^2 \frac{\sigma_n^2}{2} - \rho_n \,, \tag{26}$$

$g(z, \bar{t}, n)$ is taken into account to rewrite Formula (23), then the form of heat conduction equation can be obtained as follows:

$$-\frac{\partial g}{\partial \bar{t}} + \hat{C}_n^2 \frac{\partial^2 g}{\partial z^2} = 0, \hat{C}_n^2 = \frac{\sigma_n^2}{2} \,, \tag{27}$$

in which $n = 0, 1, \cdots, N-1$. Following initial conditions can obtained according initial condition (4):

$$\begin{cases} g(z, \bar{t}_0, 0) = Le^{-\alpha_0 z}(e^z - e^k)1_{(z \geq \delta)}, \delta = \max\{k, 0\}, \\ g(z, \bar{t}_n, n) = g(z, \bar{t}_n, n-1)\exp\{z(\alpha_{n-1} - \alpha_n) + (\beta_{n-1} - \beta_n)\bar{t}_n\}1_{(z \geq 0)} \end{cases} \tag{28}$$

in which $1 \leq n \leq N-1$. Above PDE has only one analytic solution for every time interval $\bar{t} = [\bar{t}_n, \bar{t}_{n+1}]$:

$$g(z, \bar{t}, n) = \begin{cases} L\int_0^\infty S_n(z-\xi, \bar{t} - \bar{t}_n)e^{-\alpha\xi}(e^\xi - e^k)1_{(\xi \geq \delta)} d\xi \,, \\ \int_0^\infty S_n(z-\xi, \bar{t} - \bar{t}_n) g(\xi, \bar{t}_n, n-1) e^{\{\xi \Delta\alpha_n + \Delta\beta_n \bar{t}_n\}} 1_{(\xi \geq 0)} d\xi \,, \end{cases} \tag{29}$$

In which $\Delta\alpha_n = \alpha_{n-1} - \alpha_n$, $\Delta\beta_n = \beta_{n-1} - \beta_n$. Kernel function $S_n(z, \bar{t})$ is Gaussian distribution function ($N(0, \sqrt{4\hat{C}_n^2 \bar{t}})$).

$$S_n(z, \bar{t}) = \frac{1}{\sqrt{4\pi \hat{C}_n^2 \bar{t}}} \exp(\frac{-z^2}{4\hat{C}_n^2 \bar{t}}) \,, \tag{30}$$

in which $n = 0, 1, \cdots, N-1$. According to the result obtained, the theorem of discrete barrier option pricing in monitoring date is as follows:

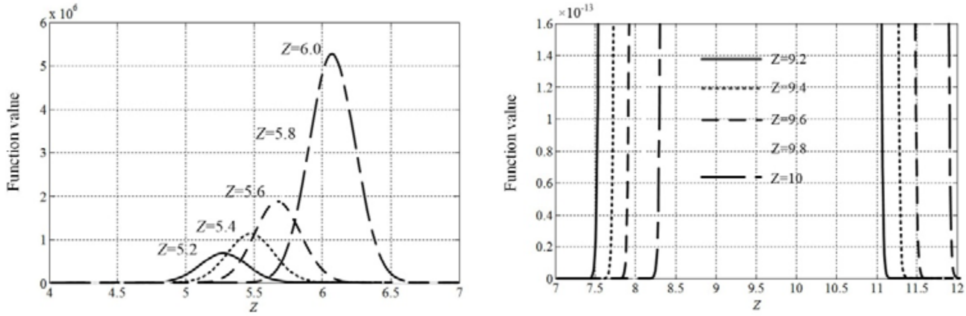
Theorem 3: when share price is x, executive price is E and barrier level is L, the barrier option pricing of discrete monitoring date $t = t_{n+1}$ is as follows:

$$P_b(x, t_{n+1}, n) = g(\ln(\frac{x}{L}), t_{n+1}, n) \exp\left\{\alpha_n \ln(\frac{x}{L}) + \beta_n t_{n+1}\right\} . \tag{31}$$

In which $n = 0, 1, \cdots, N-1$. The definitions of α_n and β_n are given in Formula (26). $g(\cdot, t_{n+1}, n)$ can be computed by using Formula (29).

3.2. Description of Romberg numerical algorithm

$g(z_0, \bar{t}_{N+1}, N)$ is computed by adopting reverse numerical iterative algorithm as the finial pricing of discrete barrier option after N monitoring dates. Because all the integral functions in Formula (29) have Gaussian distribution function $S_n(z-\xi,\tau)$, which has exponential attenuation characteristic, l_n shall be selected appropriately, the inappropriate integral in semi-infinite interval $[\max(0, z-I_n), z+l_n]$ can be approximate to the appropriate integral in interval $[\max(\delta, z-I_n), z+l_n]$, as shown in Fig. 1.



(a) Proper Integral Distribution (b) Improper Integral Distribution

Fig. 1. Integral function image of barrier options

As shown in the integral function image of discrete barrier option of monitoring point Z given in Fig. 1(a), with the increase of value Z, the maximums of these Gaussian images are increased rapidly. As shown in the integral function image of discrete barrier option of monitoring point Z given in Fig. 1(b), the field of exponential attenuation integration in some interval is inappropriate. New integral boundary is defined by using Formula (29), and following approximate definition can be computed:

$$g(z,\bar{t},n) \cong \begin{cases} L\displaystyle\int_{\max(\delta,z-l_n)}^{z+l_n} S_n(z-\xi,\bar{t}-\bar{t}_n)e^{-\alpha\xi}(e^{\xi}-e^{k})d\xi\,, \\ \displaystyle\int_{\max(0,z-l_n)}^{z+l_n} S_n(z-\zeta,\bar{t}-\bar{t}_n)g(\xi,\bar{t}_n,n-1)e^{\{\xi\Delta\alpha_n+\Delta\beta_n\bar{t}_n\}}d\xi\,. \end{cases} \tag{32}$$

In general cases, many numerical integration methods can be applied to compute above integrals. But in order to reduce the computational complexity of the function and improve the computing speed, the Romberg method is used here. Following computational process shall be paid attention:

(1) In order to compute $g(z_0,\bar{t}_N,N-1)$, it is required to know $g(\xi,\bar{t}_{N-1},N-2)$, in which $\xi\in I_{N-1}=[\max(0,z_0-l_{N-1}),z_0+l_{N-1}]$.

(2) Similarly, in the case of $z\in I_{N-1}$, in order to compute $g(z,\bar{t}_{N-1},N-2)$, it is required to know $g(\xi,\bar{t}_{N-2},N-3)$, in which $\xi\in I_{N-2}=[\max(0,z-l_{N-2}),z+l_{N-2}]$. Thus $g(\xi,\bar{t}_{N-2},N-3)$ shall be computed by following formula:

$$I_{N-2}=[\max(0,z_0-l_{N-1}-l_{N-2}),z_0+l_{N-1}+l_{N-2}]\,. \tag{33}$$

(3) In order to compute $g(z,\bar{t}_2,1)$, in which $z\in I_2$, $g(\zeta,\bar{t}_1,0)=L\int_0^{\infty}S_1(\zeta-$

$\xi, \tau)e^{-\alpha\xi}(e^{\xi} - e^{k})1_{(\xi \geq \delta)}d\xi$ shall be computed in following interval:

$$\zeta \in I_1 = [\max(0, z_0 - \sum_{i=1}^{N-1} l_i), z_0 + \sum_{i=1}^{N-1} l_i]\,. \tag{34}$$

In Formula (32), compute the integral in the case of variable $\xi = \ln(\bar{x}/L)$. In order to evaluate $g(z, \bar{t}_n, n-1)$ in nth step, it shall consider how to select appropriate U as upper bound in below time interval.

$$I_n = \begin{cases} \max(\delta, z_0 - \sum_{i=1}^{N-1} l_i), \max(z_0 + \sum_{i=1}^{N-1} l_i, U), n = 0 \\ \max(0, z_0 - \sum_{i=1}^{N-1} l_i), \max(z_0 + \sum_{i=1}^{N-1} l_i, U), n \neq 0 \end{cases} \tag{35}$$

Thus relevant algorithm can be expressed as the form of following pseudo-code 1:

pseudo-code 1 Barrier option pricing with n discrete monitoring dates

Input: positive integer $m \in \mathcal{N}$, number of iteration steps $N \in \mathcal{N}$, segmented interval I_i; SKOSPropertyAssertionSet and the base ontology

Output: option price $X \in \mathcal{R}^+$;

$Step \leftarrow 1$;
$numnode_1 \leftarrow 2^m.Ceil(length(I_1)) + 1$;
$h \rightarrow length(I_1)/numnode_1$;
for $i = 0 : numnode_1$ **do**
    $\xi_i \leftarrow i.h$;
end for
for $i = 0 : numnode_1$ **do**
    Compute $g(\xi_i, \bar{t}_1, 0)$ by Gaussian quadrature;
end for
for $step = 2 : N - 1$ **do**
    $numnode_{step} \leftarrow 2^m.Ceil(length(I_{step})) + 1$;
    $h \leftarrow length(I_{step})/numnode_{step}$;
    for $i = 0 : numnode_{step}$ **do**
        $\xi_i \leftarrow i.h$;
    end for
    for $i = 0 : numnode_{step}$ **do**
        Compute $g(\xi_i, \bar{t}_{Step}, step - 1)$ by using Romberg;
    end for
end for
Compute $X \leftarrow g(z_0, \bar{t}_N, N - 1)$ by using Romberg.

4. Analysis of numerical computation result

A study case of European call barrier option, the result variance and its index reduced obtained by different simulation algorithms are contrasted with each other. See Literature [14, 15] for considering the down-and-out discrete barrier option problem as shown in Table 1. The Constant parameters used in the case are: share price 100, executive price100, $\rho = 0.1$, $\sigma = 0.3$, expiration date $T = 0.2$. The algorithms selected for contrast are: recursion integral (RI), continuous monitoring formula (CC), ternary tree (TT), Simpson quadrature (SQ), Monte Carlo (MC) and analytic solution (AS).

Table 1. Pricing of barrier option contracts

N	L	Proposed algorithm	RI	TT	SQ	MC	AS	CC
5	89	6.28075513	6.2763	6.281	6.2809	6.28092	6.28076	6.284
5	95	5.67110494	5.6667	5.671	5.6712	5.67124	5.67111	5.646
5	97	5.16724501	5.1628	5.167	5.1675	5.16739	5.16725	5.028
5	99	4.48917224	4.4848	4.489	4.4894	4.48931	4.48917	4.050
25	89	6.20979224	6.2003	6.210	6.2101	6.21059	6.20995	6.210
25	95	5.08124991	5.0719	5.081	5.0815	5.08203	5.08142	5.084
25	97	4.11594901	4.1064	4.115	4.1160	4.11621	4.11582	4.113
25	99	2.81259931	2.8036	2.812	2.8128	2.81261	2.81244	2.673
Average of computation time		5.346	9.752	10.492	9.817	9.184	8.941	5.639

It can be known from Table 1 that the computation result of proposed algorithm is closer to the result of analytic solution than the selected comparing algorithm in case 1, which means the proposed algorithm is provided with higher accuracy of option model pricing, and reflects the effectiveness of proposed method. From the perspective of computation time average, the computation time used by proposed algorithm is relatively less, the computation time used by continuous monitoring formula is least, but its computational accuracy is lowest, which reflects the higher computational efficiency of proposed algorithm.

The parameter comparison is refined by Monte Carlo (MC) simulation. The relevant parameters of MC option pricing are set as: $\sigma = 0.15$, $r = 0.05$, $T = 1$, $m = 50$ and $\rho = 0.1$. Select 500 simulated paths and carry out study and comparison for proposed algorithm and MC discrete barrier option pricing problem in the case of selecting different values as the exercise price of barrier value. The comparison information about the numerical computation pricing of discrete barrier option and theoretical barrier option pricing is given in Table 2.

It can be known from the barrier option pricing data given in Table 2, the accuracy of proposed barrier option pricing is higher than the accuracy of MC barrier option pricing, and the error between simulated pricing and theoretical pricing is about 15%. However, Monte Carlo method adopting different strategies, such as importance sampling, conditional expectation, antithetic variables and moment matching, is not better than proposed algorithm in accuracy.

Table 2. Monte carlo barrier option pricing

Barrier value/ U.S. dollar	Exercise price/ U.S. dollar	Theoretical value	Option price/ U.S. dollar				
			Proposed algorithm	Monte Carlo method			
				Importance sampling	Conditional expectation	Antithetic variables	Moment matching
92	100	2.693	2.691	2.633	2.254	2.483	2.437
92	105	1.625	1.615	1.435	1.576	1.441	1.673
88	96	1.286	1.293	1.557	1.154	1.141	1.119
85	90	1.009	1.104	1.025	0.832	0.875	1.002
85	105	0.121	0.118	0.104	0.141	0.109	0.139

5. Conclusion

A Romberg solution having barrier option partial differential Brownian mode of discrete time parameter is proposed in the paper. The Down-and-Out discrete barrier option problem is modeled as a geometric Brownian motion model with the parameters changing over time, and the time-independent PDE obtained is transformed into a simple heat conduction equation form to realize the model simplification and give the theorem of discrete barrier option pricing. The experimental shows that the algorithm proposed can improve the accuracy of Down-and-Out discrete barrier option pricing and reduce the computational complexity.

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